# Cavity Design Method for Injection-Molded Spur Gears

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Mold cavities of gears should be made larger than the product specification since plastics shrink when changing from a molten to a solid state. For injection molded spur gears, two design methods for the compensation of shrinkage are widely used. One is the module correction method and the other is the pressure angle correction method. Both methods are based on the assumption that shrinkage occurs toward the center of a molded gear. This paper deals with the shrinkage rate and proposes a method of designing gear cavity derived from the measured shrinkage rates which govern the outside diameter, the tooth depth and the tooth thickness of a molded gear. The proposed method imposes no restriction on the shrinkage direction and provides a cavity with all of the fundamental gear design parameters.

**Key Words** : Gear Cavity, Injection Molded Gear, Shrinkage Rate

**Nomenclature**

$c$ : Clearance of a tooth
$d$ : Diameter of the circle
$h$ : Chordal tooth depth
$m$ : Module
$s$ : Arc tooth thickness
$t$ : Chordal tooth thickness
$x$ : Profile shift coefficient
$y$ : Tooth height
$z$ : Number of teeth
$\alpha$ : Pressure angle
$\beta$ : Involute start angle
$\varepsilon$ : Shrinkage rate
$\varepsilon_a$ : Shrinkage rate of tooth thickness at the tip
$\varepsilon_f$ : Shrinkage rate of tooth thickness at the root
$\varepsilon_h$ : Shrinkage rate of tooth depth
$\varepsilon_r$ : Shrinkage rate of outside diameter
$\varepsilon_t$ : Shrinkage rate of tooth thickness at the Dimensionless tooth height
$\zeta$ : $=\beta-\text{inv }\theta=\beta-(\tan\theta-\theta)$
$\eta$ : Dimensionless tooth height of the cavity
$\theta$ : Profile angle of a point on a tooth profile

**Superscript**

$c$ : Gear cavity
$m$ : Average value

**Subscript**

$a$ : Outside circle
$b$ : Base circle
$f$ : Root circle
$i$ : Null variables
$n$ : The number of the measuring points
$p$ : Pitch circle

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## 1. Introduction

In injection molded gears, it is difficult to get an accurate tooth profile, because plastics shrink during the cooling step. If the cavity of a spur gear simply has oversize configuration, the profile of the molded gear results in a serious involute error(Adams, 1986).

Cavities for injection molded gears are generally considered as enlarged gears. For injection molded spur gears, two cavity design methods are widely used to allow for shrinkage. One is the module correction method that lays emphasis on the shrinkage of tooth size, and the other is the pressure angle correction method that gives priority to tooth profile(JPSE, 1995). Both methods are based on the assumption that shrinkage occurs toward the center of a molded gear. Shrinkage, however, depends on the direction of the flow of polymer materials. In many instances, shrinkage will differ when it is parallel or perpendicular to the flow(Mckinlay and Pierson, 1994). Thus, above-mentioned methods do not provide a satisfactory solution to the shrinkage problem.

In injection molded gears, the tooth thickness is one of the major factors that must be considered in the cavity design. Often, the tooth thickness of plastic gears are specially designed in order to improve strength and wear resistance. For example, when a plastic gear meshes with a metal gear, the plastic gear can be designed to have thicker tooth than the metal gear. On the contrary, from the view point of backlash, gear teeth thinning is often required since plastic gear pairs have more backlash than metal gear pairs.

In this paper, we introduce a test cavity with gear teeth and define shrinkage rates of outside diameter, tooth depth and tooth thickness, which are based on the measured data of sample gears molded out of the test cavity. From these shrinkage rates, we derive a unified design method of a cavity. The new cavity design method is intended to correct all of the fundamental gear design parameters.

## 2. Review of Cavity Design Methods

Both the module correction and the pressure angle correction methods(2, 4) are based on the fact that the base circle of a gear cavity changes to that of molded gear during the cooling step. The base diameter of the molded gear $d_b$ is expressed in terms of its module and pressure angle, and that of the gear cavity $d_b^c$ can be described in the same manner.

$$d_b = m\ z\cos\alpha \tag{1}$$

$$d_b^c = m^c z\cos\alpha^c \tag{2}$$

where $m$ is the module, $z$ is the number of teeth and $\alpha$ is the pressure angle. The superscript c denotes the gear cavity throughout this paper.

When the molded gear solidifies and becomes stable, the base diameter is changed from $d_b^c$ to $d_b$. This can be expressed as

$$\varepsilon = \frac{d_b^c - d_b}{d_b^c} \tag{3}$$

where $\varepsilon$ is the shrinkage rate.

Substituting Eqs. (1) and (2) into Eq. (3) yields

$$\varepsilon = 1 - \frac{m\cos\alpha}{m^c\cos\alpha^c} \tag{4}$$

Although the ranges of shrinkage rates of common polymer materials are generally known, shrinkage rates are usually measured and used for the cavity design to take into consideration the influence of the injection conditions.

In Eq. (4), the module and the pressure angle of the cavity are unknowns, so they cannot be determined uniquely. In the module correction method, the pressure angle is assumed to be a constant during the cooling step. Then, Eq. (4) reduces to

$$m^c = \frac{m}{1-\varepsilon} \tag{5}$$

On the other hand, in the pressure angle correction method, the module is assumed to be a constant. The pressure angle correction method is a little more complicate than the module correction method since the radial shrinkage as well as the change of the pressure angle must be considered. The formulas for the pressure angle correction method are

$$\cos\alpha^c = \frac{\cos\alpha}{1-\varepsilon} \tag{6}$$

$$x^c = \frac{1}{\tan\alpha^c}\left[x\tan\alpha + \frac{z}{2}(\mathrm{inv}\alpha - \mathrm{inv}\alpha^c)\right] \tag{7}$$

where $x$ is the profile shift coefficient. The correct profile shift coefficient $x^c$ is used for compensation of the radial shrinkage. Equation (7) is derived from the assumption that the base circle shrinks in the radial direction.

The correct parameters of the pressure angle correction method are different from those of the module correction method. Both methods, however, generate the same base circles because Eq. (4) is valid for both cases. It follows that the tooth profile of the pressure angle correction cavity is the same as that of the module correction cavity. The assumptions of both methods are that the shrinkage is uniform and occurs in the direction of the molded gear center. In practice, the direction of shrinkage may not be toward the gear center, and the shrinkage can be different along the tooth height of the molded gear. In order to make more precise injection molded gears, a detailed analysis of the shrinkage of a tooth is needed.

## 3. Shrinkage Rates of Injection Molded Gears

We introduce a test cavity to measure the shrinking properties of an injection-molded gear. In the test cavity, the full number of teeth is not necessary and the cavity with a few teeth can fulfill the purpose of the measurement. Generally, wire-cutting machine, manufactures gear cavities, so it is possible to put several teeth in the test cavity at any location.

A tooth in the test cavity must have its counterpart on the opposite side for measurement of outside diameters of sample gears molded out of the cavity. In addition, teeth must be designed with no round fillet to identify the reference line in the measurement of tooth heights, as shown in Fig. 1. Thus, when the root circle is larger than the base circle, the tooth has an involute profile only, and in the reverse case the tooth profile is composed of an involute curve and a line. In the combined profile, the line connects the involute start point and the root circle and its direction is toward the gear center.

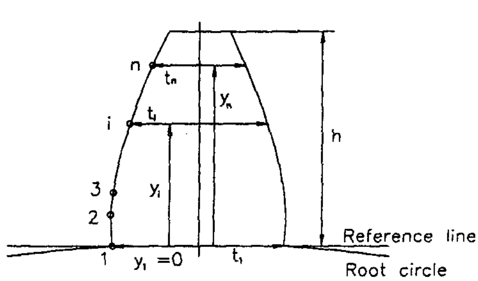


**Fig. 1** Measurement of tooth heights and thicknesses

In order to investigate the shrinking properties of a tooth, we measure the outside diameters, tooth depths and chordal tooth thicknesses along tooth heights. Let the average of the outside diameters and the chordal tooth depths of sample gears be $d_a^m$ and $h^m$, respectively. Then we can define the shrinkage rates of outside diameter $\varepsilon_r$ and of tooth depth $\varepsilon_h$ as

$$\varepsilon_r = \frac{d_a^c - d_a^m}{d_a^c} \tag{8}$$

$$\varepsilon_h = \frac{h^c - h^m}{h^c} \tag{9}$$

where $d_a^c$ is the outside diameter and $h^c$ is the chordal tooth depth of the cavity, which are given from the geometry of the test cavity.

From the measured data of the tooth thickness at the tooth height $y_i$, as shown in Fig. 1, we calculate the average tooth thickness $t_i^m$. The chordal tooth depth of each tooth varies, but we neglect this variation and define a dimensionless tooth height as $\eta_i = y_i/h^m$. Then the shrinkage rate of tooth thickness $\varepsilon_t(\eta_i)$ at the dimensionless tooth height can be calculated as

$$\varepsilon_t(\eta_i) = \frac{t_i^c - t_i^m}{t_i^c}, \ i=1, \cdots, n \tag{10}$$

where $t_i^c$ is the tooth thickness of the test cavity at height $\eta_i$ and $n$ is the number of the measuement points.

Linearizing the measured data of Eq. (10), we obtain

$$\varepsilon_t = \varepsilon_f + (\varepsilon_a - \varepsilon_f)\,\eta \tag{11}$$

where, from the geometrical point of view, $\varepsilon_a$ and $\varepsilon_f$ denote the shrinkage rates of tooth thickness at the tip and at the root, respectively.

## 4. Cavity Design Method

We apply the above mentioned shrinkage rates to the cavity design. In this section, the design parameters of a molded gear such as the module $m$, the number of teeth $z$, the pressure angle $\alpha$, the profile shift coefficient $x$, the decrement of tooth thickness $j$, and the clearance $c$ are all given values. Therefore, we can calculate the complete gear dimensions from gear formulas (5, 6). General gear formulas are not described here.

The shrinkage rate of outside diameter $\varepsilon_r$ gives the outside diameter of the cavity $d_a^c$ :

$$d_a^c = \frac{d_a}{1-\varepsilon_r} \tag{12}$$

This equation can be generalized as follows. Assuming that the shrinkage in the tooth height is linear, we can apply the shrinkage rate of tooth depth $\varepsilon_h$ to the relation between these two corresponding circles. The result is

$$\varepsilon_h = \frac{(d_a^c - d_i^c) - (d_a - d_i)}{(d_a^c - d_i^c)} \tag{13}$$

where $d_i$ and $d_i^c$ denote the diameter of the circle in the molded gear and corresponding cavity, respectively.

Substituting Eq. (12) into Eq. (13) yields

$$d_i^c = \frac{\varepsilon_r - \varepsilon_h}{1-\varepsilon_h} d_a^c + \frac{1}{1-\varepsilon_h} d_i \tag{14}$$

Then we can calculate the pitch diameter of the gear cavity $d_p^c$ by substituting that of the molded gear $d_p$ into Eq. (14). Once the correct outside diameter and pitch diameter are obtained, the correct module $m^c$ and profile shift coefficient $x^c$ are calculated as

$$m^c = \frac{d_p^c}{z} \tag{15}$$

$$x^c = \frac{d_a^c - d_p^c}{2m^c} - 1 \tag{16}$$

When the shrinkage rate of outside diameter $\varepsilon_r$ is equal to that of depth $\varepsilon_h$, Eq. (15) reduces to Eq. (5), and the profile shift coefficient remains the same.

In order to determine the tooth profile of the cavity, we apply the shrinkage rate of tooth thickness to the design. The relationship of chordal tooth thickness between the molded gear and the cavity can be obtained with the following procedure.

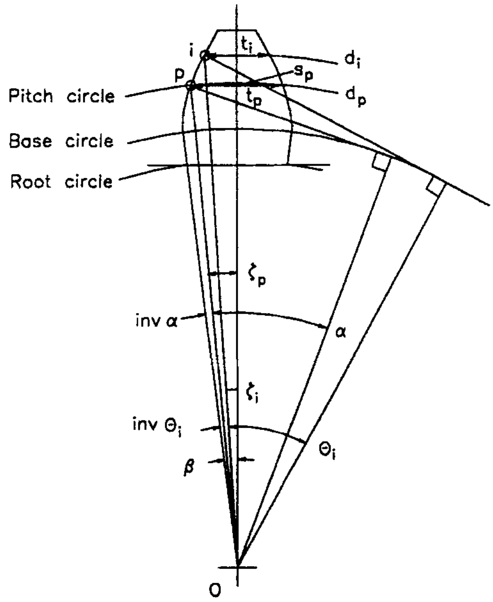


**Fig. 2** Tooth geometry

From the design parameters, the arc tooth thickness of the molded gear $s_p$ at the pitch circle is calculated as

$$s_p = \frac{\pi m}{2} + 2xm\tan\alpha - j \tag{17}$$

where $j$ is the decrement of the standard tooth thickness. In fact, the sum of $j$ values of two gears in the mesh is the amount of circular backlash when the standard center distance is adopted. The negative value of $j$ means that the molded gear has thicker teeth than the standard gear. From the gear geometry shown in Fig. 2, we obtain the chordal tooth thickness $t_p$ at the pitch circle as

$$t_p = d_p \sin\zeta_p \tag{18}$$

where $\zeta_p = s_p/d_p$. The involute start angle $\beta$ is given by

$$\beta = \mathrm{inv}\alpha + \zeta_p \tag{19}$$

The chordal tooth thickness $t_i$ and the tooth height $y_i$ at the diameter $d_i$ are calculated as

$$t_i = d_i \sin\zeta_i \tag{20}$$

$$y_i = \frac{1}{2}(d_i \cos\zeta_i - d_f \cos\zeta_f) \tag{21}$$

where $\zeta_{i,f} = \beta - \mathrm{inv}\,\theta_{i,f}$ and $\theta_{i,f} = \cos^{-1}(d_b/d_{i,f})$. Here, $\theta_f$ is valid when the root diameter is larger than the base diameter and, in the reverse case, $\theta_f$ is equal to zero. Dividing the tooth height $y_i$ by the tooth depth $h$ gives the dimensionless tooth height $\eta_i$ :

$$\eta_i = \frac{y_i}{h} \tag{22}$$

where $h = (d_a - d_f \cos\zeta_f)/2$. For consistency with the measured data, we include the rise of arc to the tooth depth. Substituting $\eta_i$ into Eq. (11), we obtain the shrinkage rate of tooth thickness $\varepsilon_{ti}$. Then the chordal tooth thickness of the cavity $t_i^c$ corresponding to that of the molded gear is obtained as

$$t_i^c = \frac{t_i}{1 - \varepsilon_{ti}} \tag{23}$$

A point on an involute can be described in terms of base diameter and roll angle. Therefore, only two points on the tooth profile are needed to define a unique involute curve. For this work, we select two circles between the base circle and the tip circle of the molded gear. Let the diameters of selected circles be $d_1$ and $d_2$. Then the diameters of the corresponding circles of the cavity, $d_1^c$ and $d_2^c$, are calculated from Eq. (14) and the chordal tooth thicknesses, $t_1^c$ and $t_2^c$, are obtained from Eqs. (17)-(23). Since two points on the tooth of the cavity are given, we can determine the involute of the cavity passing through these points. Considering the variables of Fig. 2 with the superscript $c$, we can derive the following equations from the cavity geometry.

$$\mathrm{inv}\,\theta_i^c + \zeta_i^c = \beta^c, \ i = 1, 2 \tag{24}$$

where $\zeta_i^c = \sin^{-1}(t_i^c/d_i^c)$ and $\beta^c$ denotes the involute start angle of the cavity. The formula for the base circle gives

$$d_b^c = d_i^c \cos\theta_i^c, \ i = 1, 2 \tag{25}$$

Eliminating $\beta^c$ and $d_b^c$ from Eqs. (24) and (25) yields

$$\mathrm{inv}\,\theta_1^c - \mathrm{inv}\,\theta_2^c = -\zeta_1^c + \zeta_2^c \tag{26}$$

$$d_1^c \cos\theta_1^c - d_2^c \cos\theta_2^c = 0 \tag{27}$$

Solving nonlinear Eqs. (26) and (27), we obtain the values of $\theta_1^c$ and $\theta_2^c$. Then the involute start angle $\beta^c$ and the base circle $d_b^c$ are determined from Eqs. (24) and (25), respectively.

In the above mentioned facts, if we select the base circle in the molded gear domain, the base diameter and the involute start angle of the cavity can be determined straightforwardly. In this case, the proposed method is similar to the pressure angle correction method in that it concerns only the shrinkage of the base circle.

The pressure angle of the cavity $\alpha^c$ is determined from the formula for the pitch circle and the base circle, which is

$$\alpha^c = \cos^{-1}\frac{d_b^c}{d_p^c} \tag{28}$$

The arc tooth thickness of the cavity $s_p^c$ at the pitch circle is expressed as

$$s_p^c = (\beta^c - \mathrm{inv}\,\alpha^c)\, d_p^c \tag{29}$$

Referring to Eq. (17), the decrement of tooth thickness of the cavity $j^c$ is calculated as

$$j_c = \frac{\pi m^c}{2} + 2x^c m^c \tan\alpha^c - s_p^c \tag{30}$$

Finally, in the case of the standard depth, the clearance of the cavity is calculated as

$$c^c = \frac{1}{2}(d_a^c - d_f^c) - 2m^c \tag{31}$$

where $d_f^c$ is the root diameter of the cavity which can be calculated from Eq. (14).

We have obtained the correct design parameters of the cavity corresponding to the design parameters of a molded gear. According to the location of the points used to calculate the pressure angle, the proposed method yields different results. The difference, however, does not significantly affect the tooth profiles and therefore can be neglected.

## 5. Results

In order to investigate the shrinking properties of an injection molded gear, we made a test cavity and sample gears. We used the module correction method to design the test cavity with the shrinkage rate of 0.0214 mm/mm. The design parameters of the molded gear and the test cavity are

**Table 1** Design paramenters of gear, Cavity T : test cavity and Cavity U : cavity by the unified design method

| Parameters | Gear | Cavity T | Cavity U |
|---|---|---|---|
| Module, $m$ | 1.5 | 1.533 | 1.535 |
| Number of teeth, $z$ | 32 | 32 | 32 |
| Pressure angle, $\alpha$ | 20 | 20 | 20.162 |
| Profile shift coefficient, $x$ | 0 | 0 | −0.012 |
| Decrement tooth thickness, $j$ | 0.15 | 0.153 | 0.164 |
| Clearance, $c$ | 0.375 | — | 0.342 |
| Outside diameter | 51 | 52.115 | 52.160 |
| Root diameter | 44.25 | 45.218 | 45.334 |

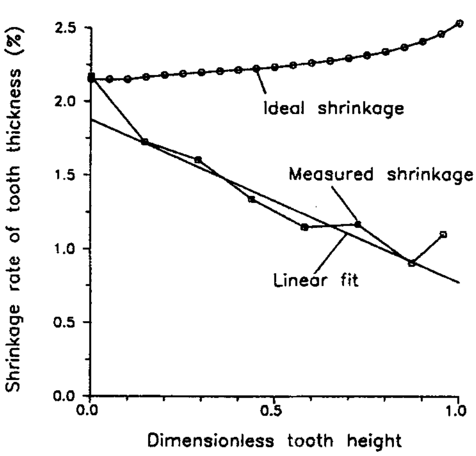


**Fig. 3** Variation of shrinkage rate with dimensionless tooth height in the test cavity

shown in Table 1. We measured 60 teeth among 20 molded gears and obtained the following shrinkage rates(SR).

-SR of outside diameter : $\varepsilon_r = 0.0222$
-SR of tooth height : $\varepsilon_h = 0.0112$
-SR of tooth thickness : $\varepsilon_f = 0.0187$, $\varepsilon_a = 0.0078$

Figure 3 shows the variation of the shrinkage

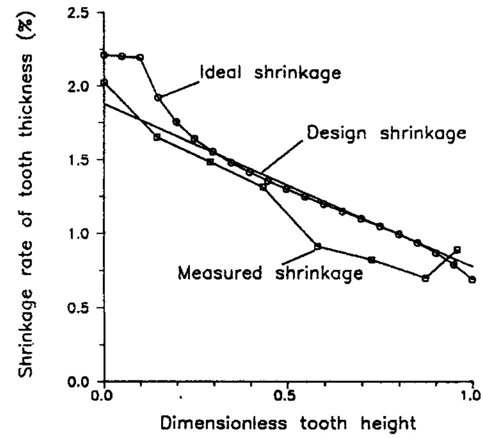


**Fig. 4** Variation of shrinkage rate with dimensionless tooth height in the Cavity $U$

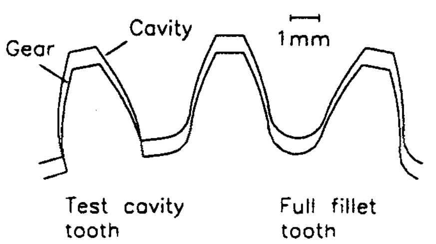


**Fig. 5** Teeth of the cavity and molded gear

rate of tooth thickness. In Fig. 3, the ideal shrinkage denotes the shrinkage rate between the tooth of the ideal cavity and that of the ideal gear. In the linearization process of the measured shrinkage rates, both end data are disregarded to get the best linear fit for the main part of the tooth profile.

Using the measured shrinkage rates, we designed Cavity $U$ by the proposed method, named the unified design method. The correct design parameters are shown in Table 1. In the unified design method, the module is first corrected to compensate the radial shrinkage and then the pressure angle is determined by the variation of the shrinkage in the tooth thickness. Figure 4 shows the analytic and experimental results of the tooth thickness shrinkage. The design shrinkage denotes the linear fit of the measured shrinkage, and it shows a good agreement with the ideal shrinkage. Thus the proposed cavity design method represents the design shrinkage in the tooth thickness quite well. We also found that the errors between the ideal shrinkage and the measured data are significantly reduced.

The proposed cavity design method corrects all of the fundamental gear design parameters, so we can easily modify the tooth profile. Figure 5 shows the teeth of the test cavity and molded gear. The left part of the figure is a tooth of the test cavity with no round fillet and the right part shows an example of a tooth with full fillet.

## 6. Conclusions

We have proposed the unified design method of the cavity for injection molded spur gears, which are derived from detailed information about the shrinkage rates. As the proposed method does not impose restrictions on the shrinkage direction, it can be applicable to the cavity design when the shrinkage variation of tooth thickness occurs. In addition, the design method provides a gear cavity with all of the fundamental gear design parameters, so commercial software can be fully used to design and manufacture a cavity for injection molded spur gears. Although some preliminary information are needed to get the detailed shrinkage rates, this method can be used to enhance the accuracy of the tooth profile of an injection molded gear.

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